$$= \frac{i\,e^{-i\frac{\pi}{2n}}}{\sin(\frac{\pi}{2n})} = \frac{i(\cos(\frac{\pi}{2n}) - i\sin(\frac{\pi}{2n}))}{\sin(\frac{\pi}{2n})} = 1 + i\frac{\cos(\frac{\pi}{2n})}{\sin(\frac{\pi}{2n})} = 1 + i\cot g(\frac{\pi}{2n})$$

Conclusion : $\sum_{k=0}^{n-1} \sin(\frac{k\pi}{n}) = \cot g(\frac{\pi}{2n})$

3°/ $S_n = \frac{1}{n}\sum_{k=0}^{n-1} \sin(\frac{k\pi}{n})$

$$\lim_{n\to+\infty} S_n = \lim_{n\to+\infty} \frac{1}{n}\cot g(\frac{\pi}{2n}) = \lim_{n\to+\infty} \frac{1}{n}\frac{\cos(\frac{\pi}{2n})}{\sin(\frac{\pi}{2n})}$$

$$= \lim_{t\to0^+} \frac{\cos(\frac{\pi}{2}t)}{\sin(\frac{\pi}{2}t)}\cdot t = \lim_{t\to0^+} \frac{\cos(\frac{\pi}{2}t)}{\frac{\sin(\frac{\pi}{2}t)}{t}} = \frac{2}{\pi} \quad \text{car} \quad \lim_{t\to0}\frac{\sin at}{t} = a$$

Solution 14 :

1°/ $\overline{Z} = \frac{e^{-i\alpha} - e^{-i\beta}}{e^{-i\alpha} + e^{-i\beta}} = \frac{\frac{1}{e^{i\alpha}} - \frac{1}{e^{i\beta}}}{\frac{1}{e^{i\alpha}} + \frac{1}{e^{i\beta}}}$

$$\overline{Z} = \frac{e^{-i\beta} - e^{i\alpha}}{e^{i\alpha}.e^{-i\beta}} \cdot \frac{e^{i\alpha}.e^{i\beta}}{e^{i\beta} + e^{i\alpha}} = \frac{e^{i\beta} - e^{i\alpha}}{e^{i\beta} + e^{i\alpha}} = -Z$$

On a : $\overline{Z} = -Z$ donc Z est un nombre imaginaire pur .

2°/ soit $W = \frac{e^{i\theta}}{e^{i\theta}+1}$

On a : $e^{i\theta} + 1 = e^{i\frac{\theta}{2}}(e^{i\frac{\theta}{2}} + e^{-i\frac{\theta}{2}}) = 2\cos\frac{\theta}{2}e^{i\frac{\theta}{2}}$

D'où $W = \frac{e^{i\theta}}{2\cos\frac{\theta}{2}.e^{i\frac{\theta}{2}}} = \frac{e^{i\frac{\theta}{2}}}{2\cos\frac{\theta}{2}} = \frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}} = \frac{1}{2} + \frac{1}{2}i\,tg(\frac{\theta}{2})$